Journal of Statistical Physics, Vol. 63, Nos. 3/4, 1991

On a Forest Fire Model with Supposed Self-Organized Criticality

Peter Grassberger[1] **and Holger Kantz**[1]

Received April 24, 1990; final September 11, 1990

We study a stochastic forest fire model introduced by P. Bak *et al.* as a model showing self-organized criticality. This model involves a growth parameter p, and the criticality is supposed to show up in the limit $p \to 0$. By simulating the model on much larger lattices, and with much smaller values of p, we find that the correlations with longest range do not show a nontrivial critical phenomenon in this limit, though we cannot rule out percolation-like critical behavior on a smaller but still divergent length scale. In contrast, the model shows nontrivial *deterministic* evolution over time scales $\gg 1/p$ in the limit $p \to 0$.

KEY WORDS: Critical phenomena; self-organized criticality; epidemics; forest fires; percolation.

1. INTRODUCTION

In a series of recent papers Bak *et al.*[1-3] have introduced the concept of "self-organized criticality" (SOC). In most conventional critical phenomena (such as the critical point in a gas–liquid transition, or a magnet at the Curie temperature), one has to fine tune a control parameter (the temperature in the above examples), in order to arrive at the critical point. Thus, if the control parameter is set at random, the system will not be critical with probability one.

In view of this, it seems hard to understand the ubiquity of $1/f$ noise (or, more correctly, of $1/f^{\alpha}$ noise, $\alpha \simeq 1$).[4] It might be that $1/f$ noise is simply not a critical phenomenon, and the observed power law is only appearent. A number of alternative explanations have indeed been proposed.[5] The other reason, advocated by Bak *et al.*, might be that

[1] Physics Department, University of Wuppertal, D-5600 Wuppertal 1, Germany.

0022-4715/91/0500-0685$06.50/0

systems are driven into a critical state *without* explicit finetuning. In this case, it would not be the experimenter, so to say, but the evolution itself which fine tunes the control parameter.

This concept is not so new indeed. There exist a number of growth models which lead to anomalous scaling laws without having fine-tuned parameters, for instance, diffusion-limited aggregation or Richardson–Eden growth.[6] Closer to SOC is the well-known model of invasion percolation.[7] In this model, each bond i (we discuss only the bond version; there exists also a completely analogous site version) is attributed a random number $r_i \in [0, 1]$ drawn from some continuous distribution. Starting from a "cluster" consisting of a single randomly chosen site, an infinite cluster is then built by adding in each time step the site reached by the perimeter bond with the smallest r_i. Here, we call a bond a "perimeter bond" if it connects a site in the cluster with a site not in the cluster. In the long-time limit, the cluster will have the statistics of an infinite percolation cluster at threshold.

Most models discussed in refs. 1–3 are indeed similar to invasion percolation, though they seem not to be in the same universality class. The only model studied by Bak *et al.* which seems unrelated to it (although, as we shall see, there is a different connection with percolation) is the forest fire" (FF) model studied in ref. 3. It is the purpose of this paper to show that this model does not indeed show SOC, but that it shows very interesting behavior nevertheless.

In the FF model, each site of a lattice can be in one of three states. It can be occupied by a tree susceptible to be burned (T), by a burning tree (B), or by ashes (A). Each tree can burn only during 1 time step, after which it turns into ash, $B \to A$. During this time step, it will put all trees in neighboring sites to fire, $BT \to AB$. These transitions both happen with probability 1. Finally, new trees can grow from the ash, $A \to T$, with small probability $p < 1$.

The model is rather unrealistic in assuming that fire is never reignited but can only propagate through the forest. A sustained fire can exist only if new trees are growing in sufficient amounts while the fire is still burning. In this respect, the model would be more applicable to the spreading of some parasite rather than a forest fire. The feature which distinguishes it from the parasite-in-an-orchard model of Hammersley[8,9] (which leads exactly to standard percolation) is the birth of new trees. If p is finite, the model is indeed in the universality class of the epidemic model with recovery,[10] which is isomorphic to directed percolation.[11]

New and interesting features arise if one takes the limit $p \to 0$. In order to have nontrivial dynamics, one has to consider the model on time scales of order $1/p$, i.e., one has to rescale time. One also has to rescale distances, as the average density of fires in a stationary state will be $\propto 1/p$. It was conjectured in ref. 3 that exactly in this limit the model shows SOC.

These claims were supported in ref. 3 by simulations on square lattices of sizes up to 100×100, and with $p \geqslant 0.01$. As noted in ref. 3, on lattices of this size the fire is extinguished after some finite time for smaller p, if one starts with statistically uniform distributions of fires and trees. Using non-uniform starting distributions, we were able to simulate the model for p down to 10^{-4}. We did this on lattices of sizes up to 4800×4800, and for times up to 8×10^4. Runs of comparable size were also made on 3- to 6-dimensional lattices.

We find no indication of a nontrivial critical behavior. Notice that the model is critical in a trivial sense: since the average density of fires scales as p, and since the spreading of fires via nearest neighbor contacts implies nonzero correlation, there must be a diverging length scale. What we claim is that the correlations on this length are trivial (for <6 dimensions), in being essentially the same as arising from straight fire fronts distances $\sim 1/p$ apart. Also, the motion of these fronts is—on the dominant length scale of the correlations—not governed by stochastic fluctuations, but is essentially deterministic. We cannot rule out that on a subdominant length scale ($\xi \ll 1/p$) there is percolation-like critical behavior, though we find no positive indication for it either.

2. SIMULATIONS AND MEAN FIELD APPROXIMATIONS

2.1. Two Dimensions

Starting with an inhomogeneous distribution of fires, two-dimensional simulations typically give spiral-like patterns as shown in Figs. 1 and 2. In Fig. 1, the distribution of trees is shown on a lattice of size 1024×1024, with $p = 0.0013$. The distribution of fires on a lattice of size 400×400 with $p = 0.0084$ is shown in Fig. 2a, while a lattice of size 4800×4800 and $p = 0.0007$ is shown in Fig. 2b. From these figures—and from many more similar ones—we see that in 2 dimensions, the fire propagates along rather regular fronts whose fluctuations decrease with decreasing p. This is of course only true if we rescale the distribution with a factor $\propto p$, as was done in Fig. 2. Otherwise said, we find that the unscaled thickness of the front increases with $1/p$ less fast than the distance between the fronts (remember that the average number of fires in a statistically stationary state scales $\propto p$, for $p \to 0$). Thus, we should expect that the characteristic length scale between regular fronts scales $\propto 1/p$, as indeed observed in Fig. 2. We thus verify the finding of ref. 3 that the distribution of fires has fractal dimension $D_f = 1$ on length scales $\xi \geqslant 1/p$, albeit in a trivial sense:

in the limit $p \to 0$, the fires occur densely along smooth lines. What happens on much smaller length scales (i.e., within the thickness of one front) is discussed later.

We should mention that the above observations refer to large but of course finite times: $T \approx 1/p$ to $10/p$. We have some indications that on even larger time scales the fronts may become fuzzier, e.g., by breaking up on small scales and creating pairs of small spirals which can then grow diffusively. But this is very slow dynamics (much slower than the rotational motion of the spirals), and we have been unable to study it systematically. Long-time runs up to 80,000 time steps were performed in two dimensions (1024^2 lattice with $p = 0.00135$ and 2048^2 lattice with $p = 5 \times 10^{-4}$) for

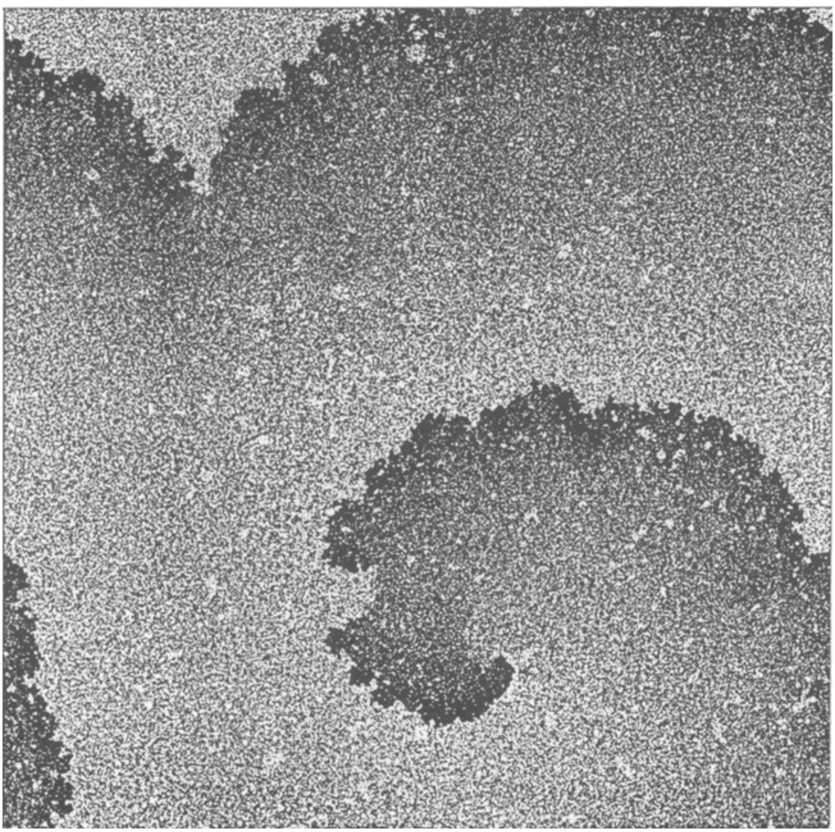

Fig. 1. Distribution of trees on a lattice of size 1024×1024, with $p = 0.0013$, after 10^4 iterations. Trees are white, ashes are black. Boundary conditions are periodic horizontally, open vertically.

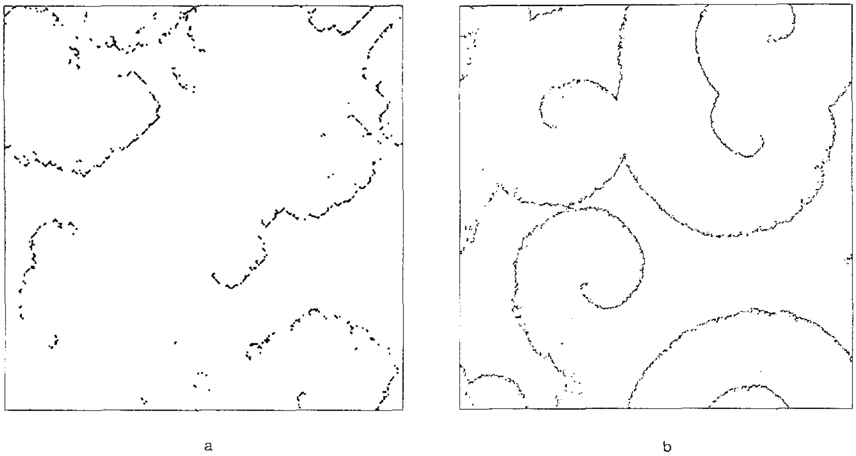

Fig. 2. Distribution of fires on two different lattices: (a) size 400×400, $p = 0.0084$, after 2112 time steps; (b) size 4800×4800, $p = 0.0007$, after 26,592 time steps.

spiral patterns, but neither the correlation dimension nor the short-time average number of fires showed significant time dependence.

The speed of propagation of a front is not fixed, for any fixed p. Assume that p is small, and that a series of sharp fronts parallel to the y axis propagate with velocities $v_k(t)$. Their positions are $x_k(t)$. The density of trees is then only a function of x and t, $\rho = \rho(x, t)$, with discontinuities at $x_k(t)$. The density of unburnt trees surviving the kth front is $\rho_k^- = \rho(x_k - \varepsilon)$, and the density just ahead of the kth front is $\rho_k^+ = \rho(x_k + \varepsilon)$ (see Fig. 3). In between, the density satisfies

$$\dot{\rho} = (1 - \rho)\, p \tag{2.1}$$

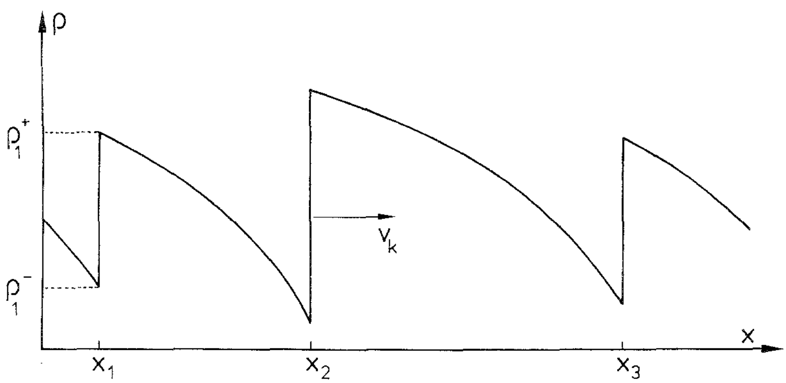

Fig. 3. Schematic density distribution in a sequence of straight flame fronts.

Neglecting correlations between the trees (i.e., making a kind of mean field assumption—a different mean field theory will be discussed in Section 2.1), it is obvious that v_k is a smoothly and monotonically increasing function $v(\rho_k^+)$ of ρ_k^+, while ρ_k^- is a smoothly and monotonically decreasing function $\rho^-(\rho_k^+)$. These two properties together with Eq. (2.1) are enough to show that any distribution of fronts will relax, on a lattice with periodic boundary conditions, toward a distribution of equidistant fronts moving with a common speed. This speed depends on the initial configuration through the average distance between the fronts, as this determines the time during which the density has grown since the last front had passed.

We simulated this on a computer for the case of only one front, i.e., the distance between adjacent fronts is the lattice length L. Boundary conditions are periodic. Elementary considerations lead then to the following relations between the density of trees $\rho^\pm$ before (resp. behind) the front, the average density of fires f, and the velocity of the front:

$$\begin{aligned} \Delta\rho \equiv \rho^+ - \rho^- &= \frac{fL}{v} \\ \ln\frac{1-\rho^-}{1-\rho^+} &= \frac{pL}{v} \end{aligned} \tag{2.2}$$

If v and ρ^- are functions of ρ^+ only, then we see that they depend on the product pL only, but not on p and L individually. In the limit $pL \to 0$, we expect that $v/pL \to \infty$ and that ρ^+ and ρ^- tend to a common value ρ_c. It follows then that $f/p \to 1 - \rho_c$.

In our simulations, we measured v, f, and $\rho^\pm$ for $L \leqslant 4800$ and for $p \geqslant 1 \times 10^{-4}$. We verified that v and fL depended on the product pL only. We did not exactly find $f \propto p$, mainly since $\Delta\rho$ is not yet small enough to justify the approximation $\ln[(1-\rho^-)/(1-\rho^+)] \propto \Delta\rho$ (ρ^- was very close to 0 in all simulations). We verified, however, Eq. (2.2), and got by extrapolation $\rho_c = 0.60 \pm 0.01$. In Fig. 4 we plot $\log v$ vs. $\log p$. We do not find as nice a scaling law as we would have hoped, but for $pL < 1$ a reasonable fit is given by

$$v \propto (pL)^{0.6 \pm 0.15} \tag{2.3}$$

This suggests very much that we are observing (site) percolation, with the fires percolating in an essentially static distribution of trees with density ρ^+. We should thus compare ρ_c with the threshold for site percolation, which is at 0.5928.[12] The difference $\Delta\rho$ would then correspond to the density of sites belonging to the infinite cluster, and would scale as $(\rho^+ - \rho_c)^\beta$. Equation (2.3) should then be compared to the relation

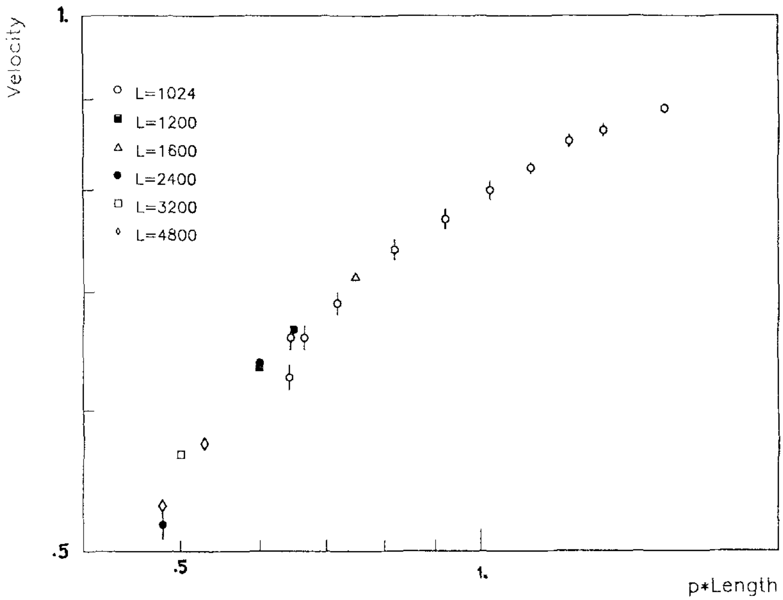

Fig. 4. Speed of flame fronts v versus growth probability $p \cdot L$ between fronts on a log-log scale. L is both the lattice size and the distance between fronts.

$v \sim (\rho^+ - \rho_c)^{\nu_t - \nu} \sim \Delta p^{(\nu_t - \nu)/\beta}$ with $(\nu_t - \nu)/\beta = 0.175 \times 36/5 = 1.26.$[9] Together with Eq. (2.2), this gives for fixed L,

$$v \sim p^{(\nu_t - \nu)/(\nu_t - \nu + \beta)} = p^{0.558}$$

in agreement with Eq. (2.3). Assuming $\rho_c = 0.5928$ as in percolation, we also found $\rho^+ - \rho_c \sim p^{2.5}$ (with rather big uncertainty), in agreement with the prediction $\rho^+ - \rho_c \sim p^{(\nu_t - \nu + \beta)^{-1}} \sim p^{3.2}$ from percolation. Finally, the correlation length ξ within the fire front should be

$$\xi \sim (\rho^+ - \rho_c)^{-\nu} \sim (pL)^{-\nu/(\nu_t - \nu + \beta)} \tag{2.4}$$

The requirement that $\xi \ll L$ thus puts a lower limit on p: $pL \gg L^{-(\nu_t - \nu + \beta)/\nu} = L^{-0.235}$. Below this limit, our picture of regularly propagating straight fronts has to break down.

Actually, the picture breaks down already for larger p, if we consider very long times. Straight fronts as discussed above are unstable against the formation of "plumes" superficially similar to the plumes rising from a boundary layer heated from below. The mechanism is of course very different in both cases. In the present case, an occasional extinction of fires in part of the front (which always happens for small p due to statistical fluctuations) leads to a region with high tree density which is then burnt from behind. After the front has passed, the region is left excessively *depleted*, and when the next front arrives the fire will again be extinguished in the same region, thus giving rise to a positive feedback. For an example of a fully developed "plume" see Fig. 5. This mechanism ultimately destroys the regular front, and temporarily gives rise to a pair of vortices. But these will ultimately also be destroyed by the formation of new plumes, and we finally get patterns similar to those seen in Figs. 1 and 2. Thus, the long-time behavior is not that of a straight front, and the above discussion cannot be used to argue that percolation is relevant in the double limit $Tp \to \infty$, $p \to 0$. In particular, it is impossible to maintain the system close to the critical point by keeping the distances between fronts so small that they proceed just marginally. This problem will be taken up again in Section 3.

Let us now come back to the more general situation shown in Figs. 1 and 2. With hindsight we see plumes also in Fig. 1, and we observed indeed that the patterns of vortices were occasionally changed by the creation of new pairs. Apart from this, we might suspect that the speed of the front is zero in the centers of the spirals. In that case, the density ρ would just be critical at these centers, and the large fluctuations there could organize the whole evolution. In this sense, the state could then still be called SOC. We claim that this is not true. Indeed, more careful investigation shows that the endpoints of the flame fronts at the centers of spirals are not fixed.

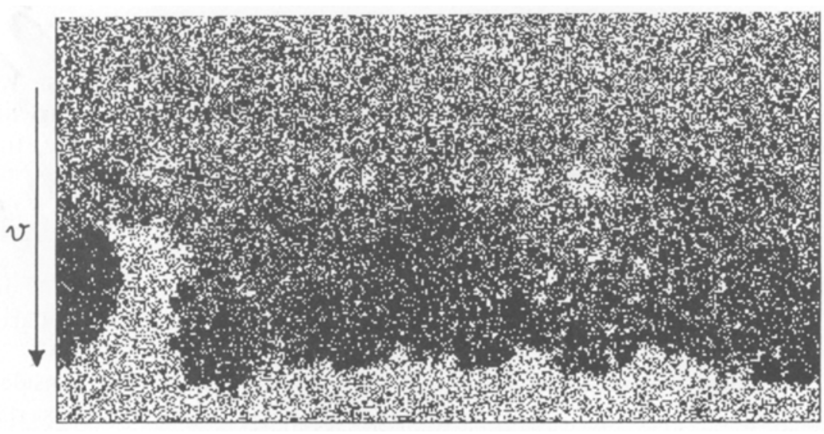

Fig. 5. Flame front in a lattice of size 4800×4800 with a single front moving downward; boundary conditions are periodic, $p = 10^{-4}$, and $t = 17{,}000$. On the left side, a "plume" is visible. After it is burnt, it will not immediately burn when the next front comes (since not enough trees will have grown yet), and a plume will form again.

Instead, for very small p they move as seen in Fig. 6: at any time, the density ρ is discontinuous along a line in the center of a spiral, and the endpoint of the flame front moves back and forth along this line.[13] Similar lines of discontinuity in the tree density which are encircled by endpoints of flame fronts arise from collisions of fronts.

If the above picture stays correct in the limit $T \cdot p \gg 1$, then the statistically stationary state in 2 dimensions is critical only in a very trivial sense. Indeed, it is essentially governed by the three deterministic equations $d\rho/dt = (1-\rho)\,p$, $v_k(t) = v(\rho_k^+(t))$, and $\rho_k^-(t) = \rho^-(\rho_k^+(t))$, with very little stochastic noise. It is not clear whether these equations alone would give a chaotic evolution, i.e., whether they show sensitive dependence on initial conditions. But we have already seen that flame fronts are unstable for very small p. When the deterministic equations are augmented by a finite amount of noise, they should give rise to the formation of plumes and thus to chaotic evolution. In this scenario, the majority of flame fronts would not move under critical conditions, since in general it takes too long for a

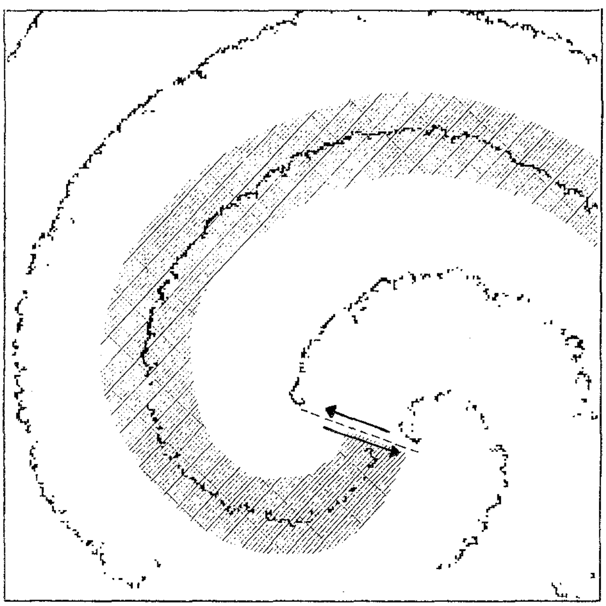

Fig. 6. Fires on part of a lattice (total size 4800×4800) at three times $t, t+\tau, t+2\tau$ separated by $\tau \approx 1/2$ of a revolution period ($p = 0.00025$, $t = 59{,}136$, $\tau = 852$). The fire front corresponding to time t is indicated by a hatched background. The edge of the fire front in the center of the spiral does not stand still, but moves back and forth along the dashed line, which for all times is a line of discontinuity of the tree density.

flame front to move into a region with critical density of trees, and the density would already be supercritical when the flames reach the region.

In an alternative scenario, the fronts could become more and more fuzzy in the limit $T \cdot p \to \infty$. In this case, ρ has to tend toward the critical density of percolation. The typical correlation length ξ would then be the one characteristic for percolation at density $|\rho - p_c|$, instead of $\xi \propto 1/p$ as in the above scenario. We shall discuss this scenario in more detail in Section 3.

2.2. Higher Dimensions

In more than 2 dimensions, similar detailed visualizations are not possible. But the higher number of neighbors suggests that we attempt a straightforward mean field approximation which should be valid at high enough dimensions. As before, we denote by ρ the density of trees, and by f we denote the density of fires. The coordination number of the lattice is denoted by N. Then the mean field equations are

$$\begin{aligned} \dot{f} &= -f + (N-1)\,\rho f \\ \dot{\rho} &= (1-\rho)\,p - (N-1)\,\rho f \end{aligned} \tag{2.5}$$

This allows for a fixed point at

$$\rho^* = \frac{1}{N-1}, \quad f^* = \frac{N-2}{N-1}\,p$$

A linear stability analysis shows that this fixed point is a stable focus, with eigenvalues

$$\lambda_\pm \simeq -(N-1)\,p/2 \pm i[(N-2)\,p]^{1/2} \tag{2.6}$$

Thus, if one starts not exactly at the fixed point, one will observe weakly damped oscillations with angular frequency

$$\omega^* = [(N-2)\,p]^{1/2} \tag{2.7}$$

For finite p, there will be stochastic fluctuations superimposed on these regular oscillations. They might even compensate the damping, thus leading to sustained nearly regular oscillations with angular frequency $\simeq \omega^*$.

We tried to reproduce these mean field solutions in numerical simulations. We started with either inhomogeneous or homogeneous initial conditions. In the latter case, the fires and trees were put randomly, with densities f^* and ρ^*. While the spirals created by inhomogeneous initial conditions had stabilized the system in 2 dimensions, this was true to a much smaller degree in higher dimensions. Here we always had the problem that the system has the tendency to make oscillations large enough so that the fires are completely extinguished. Nevertheless, for not too small p and for sufficiently large lattices, we did find configurations which were stable over sufficiently long times.

In Fig. 7 we show a typical time series obtained from a simulation in $D=4$. We see very marked oscillations which are damped at first, but which settle then at a more or less constant amplitude. Similar behavior was found in $D=3$, 5, and 6. The periods were not very dependent on the amplitudes. From Fig. 8, where the small-amplitude periods are plotted against D, we see that the values expected from Eq. (2.7) are reached very slowly. In general, the frequencies were decreasing faster than $\sqrt{p}$. For $D=3$, our results were indeed closer to $\omega \propto p$ than to $\omega \propto \sqrt{p}$.

A limited visualization of a configuration is possible in $D=3$ by making either projections of the distribution of fires or by making cuts through the distribution of trees. While the former suggested rather diffuse

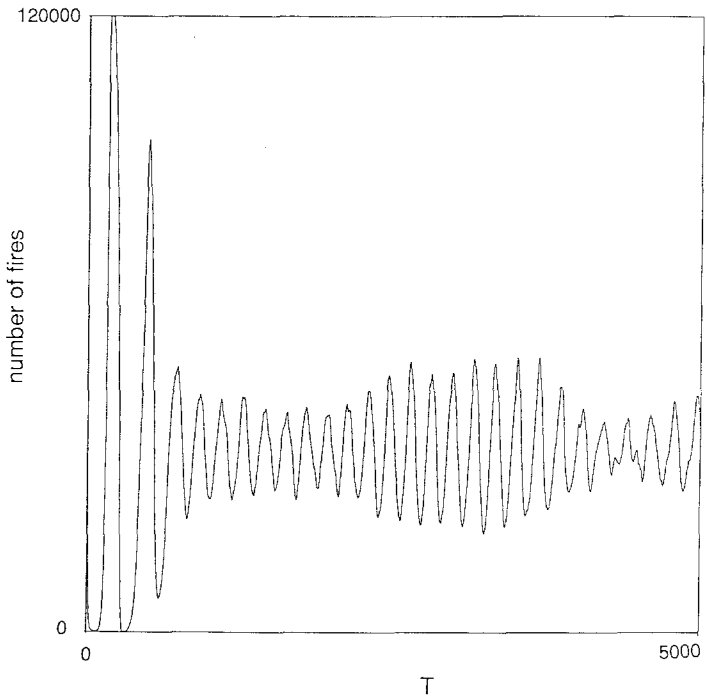

Fig. 7. Number of fires as a function of time in a $4D$ lattice of size $80^3 \times 96$, with $p=0.00085$.

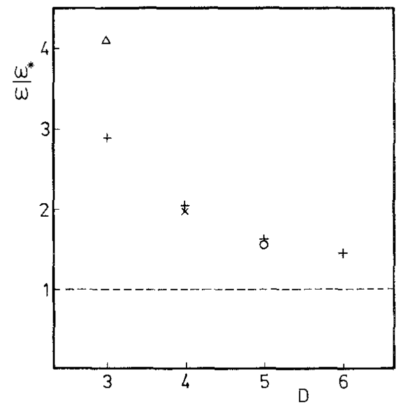

Fig. 8. Frequencies of oscillation versus dimensionality. In order to compare with Eq. (2.7), the quantity actually plotted is $\omega[(2D-2)p]^{-1/2}$. Data for different values of p are superimposed: 0.001 (+), 0.002 (Δ), 0.00085 ($\times$), 0.0012 ($\bigcirc$).

distributions, the latter indicated clearly (see Fig. 9) that fires propagate also here along regular fronts. These fronts are of course 2 dimensional, in contrast to the situation in $D=2$. This suggests that also in $D=3$ we find no critical behavior, at least on feasible time and length scales.

We checked this by estimating the fractal dimensions D_f of the fire distribution, estimated from the correlation integrals $C(r)$ (i.e., the number of pairs of fires with a distance $<r$). For fractal distributions, we expect

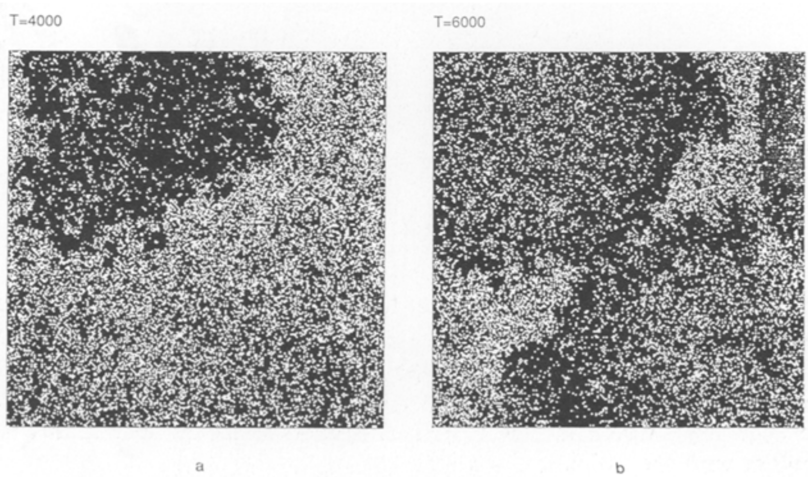

Fig. 9. Two typical cross sections through $3D$ lattices ($L=256$, $p=0.001$). Trees are white, ashes are black. Times are (a) 4000 and (b) 6000 after a homogeneous random start.

$C(r) \sim r^{D_f}$ in a range $1 \ll r \ll p^{-1/D}$ (remember that the density of fires is $\sim p$). We found this reasonably well obeyed for $D=2$ and 3, with $D_f = D-1$. We definitely can rule out the value $D_f = 2.5$ found on much smaller lattices (and with much larger p) in ref. 3. For higher D, no scaling was reached within the expected range of distances.

3. POSSIBLE RELATION TO PERCOLATION

Since flame (hyper-)fronts should become more and more fuzzy in higher dimensions, it is more likely than in $D=2$ that the true asymptotic behavior is percolation-like. More precisely, since we are for any finite p not precisely at the critical point, we could expect that we find percolation-like behavior for times and distances less than characteristic values T, ξ. Within this region, the growth of new trees could be neglected, and the spreading of fires would be as in an epidemic process without recovery (or "dynamical percolation"[9]). The above mean-field theory would be qualitatively correct for $D > 6$.

Let us first derive D_f for percolation. In the terminology of refs. 9 and 14, it is the fractal dimension of the "growth sites" and is obtained as follows. Consider a cluster of infected (burnt) sites of radius r which all come from a common ancestor, which started the cluster a time $t \sim r^{\nu - \nu_t}$ ago. This cluster will contain $N(t) \sim r^{d_f}$ sites, where $d_f = D - \beta/\nu$ is its fractal dimension. Since each site burns only for 1 time step, the number of growth sites is equal to $n(t) = dN(t)/dt \sim r^{d_f - \nu_t/\nu}$. From this we see that

$$D_f = D - \frac{\beta + \nu_t}{\nu} \qquad \text{(epidemic process)} \tag{3.1}$$

For $D=2$, 3, and 4, this gives[9,14] $D_f = 0.764$, 1.13, and 1.42, respectively. For $D \geq 6$, we find $D_f = 2$.

The fact that we found much larger fractal dimensions of fires gives a first argument that the critical behavior is not that of percolation.

Let us now estimate the ranges T and ξ. Naively we would guess that the characteristic time scale for the process is $T = 1/p$. If we are close to percolation, the corresponding length scale is then $\xi = T^{\nu/\nu_t} \sim p^{-\nu/\nu_t}$, and the fluctuations of ρ on length scales ξ should be $\Delta\rho \sim \xi^{\beta/\nu}$. But in this case, spreading would be locally isotropic (i.e., not perpendicular to smooth fronts). An upper limit for ξ would then be set by the average distance between fires, which is $\sim p^{-1/D}$. Since $\nu/\nu_t > 1/D$ for all $D \geq 2$, this is in conflict with the estimate $\xi \sim p^{-\nu/\nu_t}$.

Unfortunately, this argument against percolation is fallacious since the naive estimate $T \sim 1/p$ is wrong. T is not set by the time during which ρ

changes by a finite amount, but by times during which enough trees in a burnt region are newly grown to reach a critical density again: $T \sim \Delta\rho/p$. Close to the percolation threshold, $\Delta\rho \to 0$. If we assume percolative behavior on all length scales, then we expect by arguments similar to those in Section 2.1 that

$$\Delta\rho \sim \xi^{-\beta/\nu} \tag{3.2}$$

and we get a self-consistency relation

$$\xi \sim \left(\frac{\Delta\rho}{p}\right)^{\nu/\nu_t} \sim p^{-\nu/(\nu_t+\beta)} \tag{3.3}$$

For all $2 \leqslant D < 6$, this is still in conflict with $\xi \leqslant p^{-1/D}$, thus ruling out the scenario where percolation-like behavior dominates on all length scales.

If, on the other hand, we assume that fires propagate along fronts with distances of order $L \gg \xi$ between each other, then we have two alternating scenarios, none of which we can at present favor:

(i) $pL \to 0$ for $p \to 0$. In this case, ξ diverges according to Eq. (2.4), and fronts progress like critical percolation processes. Notice that in this case there are 2 independent diverging length scales.

(ii) $pL \to \text{const} \neq 0$. Now the average thickness of the fronts stays finite, and the model is not critical at all.

Both scenarios would explain why percolative behavior has not been observed in our simulations.

4. CONCLUSIONS

We have presented numerical results for a forest fire model— or, more realistically, a model for the spreading of an epidemic with very slow recovery. According to the claims of ref. 3, this model should display self-organized criticality. Our results suggest that this is not true. On all time and length scales feasible in our simulations (which are larger by orders of magnitude than those of ref. 3), the model showed no nontrivial critical behavior at all. In particular, in 2 dimensions flame fronts are surprisingly smooth (giving thus a fractal dimension $D_f = 1$ for the distribution of fires), and their motion is not fluctuation-dominated over distances comparable to the correlation length. Instead, one finds essentially deterministic propagation of the flame fronts on this length scale. Superimposed on it is stochastic noise whose amplitude seems to vanish in the relevant limit of vanishing growth rate p for new trees. The resulting motion is extremely complicated and not yet fully understood.

In 3 dimensions, the correlation dimension of the fires is $D_f = 2.0 \pm 0.2$, different from the value found 2.5 found in ref. 3. We argued that if there were essential finite-time or finite-size corrections, then the correct value of D_f should be even lower.

In ref. 3, the agreement of D_f with a supposed value of the fractal dimension of eddies in fully developed turbulence has led the authors to speculate on a possible relationship between the present model in the limit $p \to 0$ and hydrodynamic turbulence. We see no such connection. While the dynamics in 2 and 3 dimensions is characterized by nearly deterministic evolution of macroscopic fronts, in higher dimensions we approach a mean-field behavior governed by rate equations allowing for (nearly) periodic oscillations around a fixed point. For $D \geqslant 6$, this fixed point is just the mean-field behavior of percolation.

If the true asymptotic behavior is critical (contrary to what is suggested by our simulations), then it should be in the same universality class as percolation also for $D < 6$. More precisely, it should then coincide on time scales $T \ll 1/p$ and on length scales $< T^{\nu_t - \nu}$ with "dynamical" percolation in the sense of ref. 9, or with the "epidemic process with removal." If this is true, we expect critical behavior to set in only very late (in time, in p, and on very large lattices), and we would expect the critical correlation length not to be the dominant diverging length scale. This would explain why it is de facto unobservable.

ACKNOWLEDGMENT

One of us (P.G.) wants to thank Dr. Mario Markus for very instructive discussions, and for pointing out ref. 13.

REFERENCES

1. P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. Lett.* **59**:381 (1987); *Phys. Rev. A* **38**:364 (1988); C. Tang and P. Bak, *Phys. Rev. Lett.* **60**:2347 (1988); *J. Stat. Phys.* **51**:797 (1988).
2. P. Bak and K. Chen, *Physica D* **38**:5 (1989).
3. P. Bak, K. Chen, and C. Tang, preprint (1988).
4. P. Dutta P. M. Horn, *Rev. Mod. Phys.* **53**:497 (1981); M. B. Weissman, *Rev. Mod. Phys.* **60**:537 (1988).
5. E. Marinari, G. Parisi, D. Ruelle, and P. Windey, *Commun. Math. Phys.* **81**:1 (1981); E. W. Montroll and M. F. Shlesinger, *Proc. Natl. Acad. Sci. USA* **79**:3380 (1982).
6. D. Wilkinson and J. F. Willemsen, *J. Phys. A* **16**:3365 (1983).
7. T. Vicsek, *Fractal Growth Phenomena* (World Scientific, Singapore, 1989).
8. S. Broadbent and J. Hammersley, *Math. Proc. Camb. Phil. Soc.* **53**:629 (1957).

9. P. Grassberger, *Math. Biosci.* **62**:157 (1983); *J. Phys. A* **18**:L215 (1985).
10. T. M. Liggett, *Interacting Particle Systems* (Springer, New York 1985).
11. P. Grassberger, in *Fractals in Physics*, L. Pietronero and E. Tosatti, eds. (North-Holland, Amsterdam, 1986), p. 273.
12. D. Stauffer, *Introduction to Percolation Theory* (Taylor and Francis, London, 1985).
13. N. Wiener and A. Rosenblueth, *Arch. Inst. Cardiol. Mex.* **16**:205 (1946).
14. P. Grassberger, *J. Phys. A* **19**:1681 (1986).